A New Time Domain Near Field to Far Field Transformation for FDTD in Two Dimensions

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Abstract **An algorithm of 2-D near field to far field transformation for the finite difference time-domain (FDTD) method is proposed in this paper. The transient far field response of a two-dimensional structure is obtained straightforwardly in the time domain without Fourier transformation. The abnormal integral for time in 2-D time domain Green function is transformed to a normal form by means of analytic method. Besides, by use of quantifying the positions of equivalent sources and arranging the time series, the calculations of the time integrals of equivalent sources are converted to the calculation of matrix-vector multiplying. A good agreement between both results has been observed.**

I. Introduction

The Finite difference time domain (FDTD) method, first introduced by Yee [1], is a powerful, robust, and popular modeling algorithm based on the direct numerical solution of Maxwell's equations in the differential, time domain form. Usually the far field should be deduced from the near field due to the finite dimension of the problem. Basically, two methods for near field to far field transformation have been successfully used in FDTD method for two- and three-dimensional problems. One is in the frequency domain [2,3], another is in the time domain for three-dimensional transient calculations [4,5]. Because the evaluation of the time domain Green function in two dimensions is not straightforward, a mixed frequency-time domain algorithm was proposed in Reference [6], which involves two Fourier transformations to obtain the time domain response. In Reference [7], a method was proposed to directly calculate the transient far-field response of a two-dimensional structure. However, a fourth-order interpolating polynomial must be introduced to calculate the abnormal time integral in the time domain Green function.

This paper presents a new approach to directly compute the transient far-field response of a two-dimensional structure. By means of analytic transformation, the abnormal time integral is converted to a normal time integral, no need of a fourth-order interpolating polynomial. This increases the calculation speed and accuracy. By use of quantifying the positions of equivalent sources and arranging the time series, the calculations of the time integrals of equivalent sources are converted to the calculation of matrix-vector multiplying. The memory of matrix can be compressed to that of vector. At last, the transient far-field response can be obtained more quickly and exactly. That would allow calculating the mutual coupling for the multiple regions problems of two-dimensional structures in FDTD method.

II. Formulation

In the near-zone of a scatterer, the equivalence principle allows the substitution of the actual sources of these fields by a set of equivalent electric and magnetic surface current densities. These currents are expressed as,

$$J^s(\bar{\rho}',t) = \hat{n} \times \bar{H}^s(\bar{\rho}',t) \qquad (1)$$

$$M^s(\bar{\rho}',t) = -\hat{n} \times E^s(\bar{\rho}',t) \qquad (2)$$

where $\hat{n}$ is a outward unit vector normal to the surface. For simplicity, the equivalent electric current source $J_z^s(t)$ of TM mode is only discussed here. The case of magnetic current source of TM mode and the case of TE mode are similar. The transient far-field response of the electric current source is expressed as

$$E_z(\bar{\rho},t) = -\int_{l'}\int_{-\infty}^{+\infty} \bar{G}(\bar{\rho},\bar{\rho}',t,t')\mu \frac{\partial J_z(\bar{\rho}',t')}{\partial t'} dt'\, dl' \qquad (3)$$

and

$$G(\bar{\rho},\bar{\rho}',t,t') = \frac{c}{2\pi\sqrt{c^2(t-t')^2 - |\bar{\rho}-\bar{\rho}'|^2}} H\left[c(t-t') - |\bar{\rho}-\bar{\rho}'|\right] \qquad (4)$$

where, c is the speed of light, $H\left[c(t-t') - |\bar{\rho}-\bar{\rho}'|\right]$ is the Heaviside function. When the excitation of electric current source is a signal starting from zero time, the transient response can be expressed as

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$$E_z(\bar{\rho},t) = -\frac{\partial A_z(\bar{\rho},t)}{\partial t}$$

and

$$A_z(\bar{\rho},t) = -\int_l \int^{t-\frac{|\bar{\rho}-\bar{\rho}'|}{c}} \frac{\mu c}{2\pi\sqrt{c^2(t-t')^2 - |\bar{\rho}-\bar{\rho}'|^2}} J_z(\bar{\rho}',t')dt'dl' \quad (5)$$

Obviously, in equation (5), when $t' = t - |\bar{\rho}-\bar{\rho}'|/c$, the time integral is an abnormal integral. So the integral for time not only exists in the free space Green's function of 2-D, but also is abnormal. It is obvious that the problem of 2-D structures is much more difficult than that of 3-D structures.

In FDTD iterative calculation, although the equivalent electric and magnetic currents are arbitrary, because Δt is small enough, they can be regarded as linear cases. In some Δt, suppose $\tau = t - t'$, the equivalent electric current can be written as,

$$J_z(\tau) = J_z(n\Delta t) + \frac{J_z((n+1)\Delta t) - J_z(n\Delta t)}{\Delta t} \cdot (\tau - \rho_0/c - n\Delta t)$$

$$n\Delta t \le \tau - \rho_0/c < (n+1)\Delta t \quad (6)$$

Substitute equation (6) into (5), the equation (5) can be rewritten as

$$A_z(\rho_0,t) = \int_l \sum_{n=0} \int_{\frac{\rho_0}{c}+n\cdot\Delta t}^{\frac{\rho_0}{c}+(n+1)\cdot\Delta t} \frac{\mu c}{2\pi\sqrt{c^2\tau^2 - \rho_0^2}} \cdot [G(n)\tau + H(n)]d\tau dl'$$

$$= \int_l \frac{\mu c}{2\pi} \sum_{n=0} \left[\frac{G(n)}{c^2}\sqrt{c^2\tau^2 - \rho_0^2}\Bigg|_{\frac{\rho_0}{c}+n\cdot\Delta t}^{\frac{\rho_0}{c}+(n+1)\cdot\Delta t} + \frac{H(n)}{c}\ln\left(c\tau + \sqrt{c^2\tau^2 - \rho_0^2}\right)\Bigg|_{\frac{\rho_0}{c}+n\cdot\Delta t}^{\frac{\rho_0}{c}+(n+1)\cdot\Delta t} \right] dl' \quad (7)$$

where, $\tau = t - t'$

$$\rho_0 = |\bar{\rho}-\bar{\rho}'|$$

$$G(n) = [J((n+1)\Delta t) - J(n\Delta t)]/\Delta t$$

$$H(n) = J(n\Delta t) - G(n)\cdot(\rho_0/c + n\cdot\Delta t)$$

As shown in equation (7), by means of analytic method, the abnormal time integral has been transformed to a normal integral. The calculation of magnetic current sources is similar with the electric current sources.

For stating clearly, we use equation (5) to explain how the calculations of the time integrals of equivalent sources in equivalent enclosing line are converted to the calculation of matrix-vector multiplying. According to the equation (5), the calculation for the transient far-field response of singular line source can be expressed as a calculation of matrix-vector multiplying,

$$\begin{bmatrix} A_z(t) \\ A_z(t+dt) \\ \cdots \\ \cdots \\ \cdots \end{bmatrix} = \begin{bmatrix} J(0) & \cdots & \cdots & \cdots & \cdots \\ J(1) & J(0) & \cdots & \cdots & \cdots \\ J(2) & J(1) & J(0) & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{bmatrix} \cdot \begin{bmatrix} G_1 \\ G_2 \\ \cdots \\ \cdots \\ \cdots \end{bmatrix} \quad (8)$$

where G_1, G_2, $\cdots$, are the values in equation (5) except for electric current. When the other currents perform above matrix-vector multiplying calculation, their **[G]** matrixes are the same. All the source points are projected to the direction of from the center point of equivalent sources enclosing line to far field point (see Fig.1). According to the projection distances of the source points, the time series of sources can be obtained. The source electric currents are put into **[J]** matrix only by plus calculation according the time series. Because the plus calculation is very fast, basically, the calculation of the time integrals of equivalent sources is converted to the calculation of a matrix-vector multiplying. In fact, in calculation process, the memory of **[J]** matrix can be compressed to the memory of a vector. By means of these steps, the calculation time and memory are largely decreased.

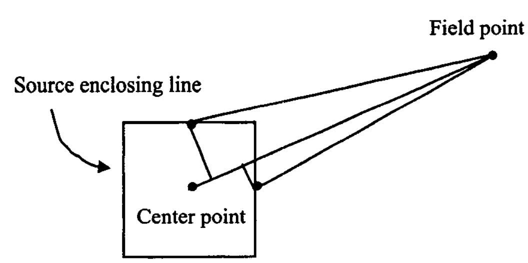

Fig.1. The projection of the source points

III. Results

To verify the algorithm, we present results for the far field scattered by two mixed cylinders. A Gaussian pulse TM polarized illuminates the cylinder. In Fig.2, a circular cylinder and a square cylinder compose the mixed cylinder. The radius of the circular cylinder is 22mm, $\varepsilon_r = 3$. The edge length of the square cylinder is 12mm, $\varepsilon_r = 10$. The incident direction is 45 degree. We calculate the wide band mono-RCS by use of frequency-domain method [2,3] and our time-domain method, respectively. In Fig. 3, the edge lengths of the exterior square 44mm, $\varepsilon_r = 5$. The edge length of the interior square cylinder is 12mm, $\varepsilon_r = 15$. The incident direction is 0 degree. Excellent agreements between two transformation methods results are found both in the two figures. Besides, the calculation speed of our method is faster about six times than that of frequency-domain transformation method.

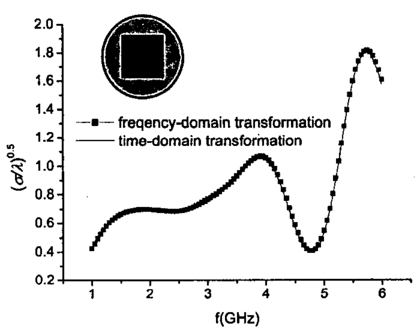

Fig.2. The mixed cylinder RCS

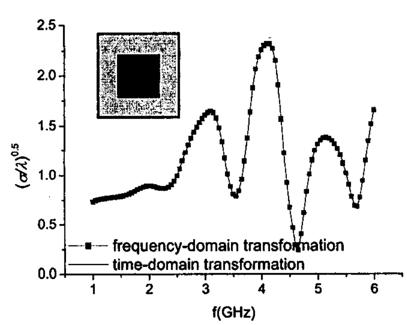

Fig.3. The mixed square cylinder RCS

Furthermore, with this algorithm, we can solve this kind of problems, multi-cylinders under a TE or TM mode illuminating. Because the transient field responses are obtained straightforwardly in the time domain, the mutual coupling between two-dimensional structures can regarded as the cylindrical wave incidence

In Fig.4, two square cylinders are apart from 600mm according their center points. Both of cylinders are the same. The edge length of square is 44mm, $\varepsilon_r = 3$. They are placed as shown in Fig. 4. The incident signal is TM wave. The incident direction is 0 degree. The RCS pattern is calculated by use FDTD method and moment method (MM). The frequency is 1GHz. An excellent agreement between two methods results is observed in Fig.4.

In Fig.5 and Fig.6, the same two square cylinders are apart from 400mm and 300mm according their center points, respectively. Excellent agreements between two methods results are also observed from the Fig.5 and Fig.6.

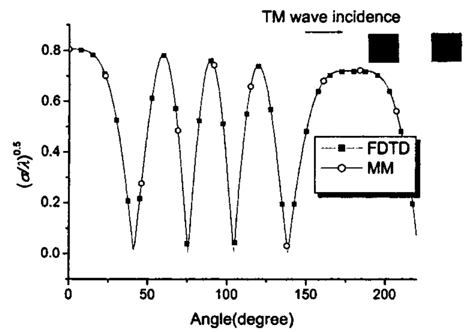

Fig.4. The bistatic RCS of two dielectric square cylinders apart from 600mm

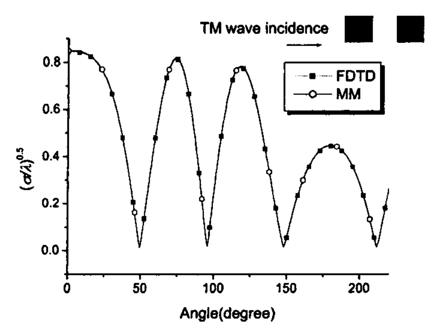

Fig.5. The bistatic RCS of two dielectric square cylinders apart from 400mm

IV. CONCLUSION

In this paper, we have proposed an algorithm to calculate the transient far-field response of a two-dimensional structure straightforwardly in the time domain. By means of analytic transformation, the abnormal time integral is converted to a normal time integral without interpolating [7]. The time and memory of calculation are considerably decreased than frequency-domain transformation method, when the wide band RCS is calculated. The algorithm would allow calculating the multiple regions problems of two-dimensional structures using FDTD method.

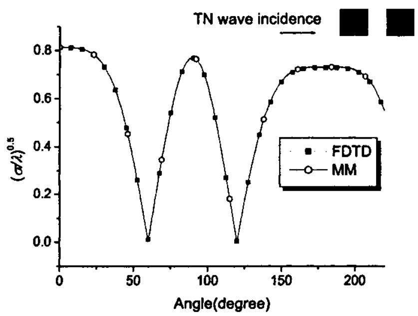

Fig.6. The bistatic RCS of two dielectric square cylinders apart from 300mm

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